UNCERTAINTIES OF SPECTRAL RESPONSIVITY MEASUREMENTS

G. P. Eppeldauer, G. Sauter[1], and J. L. Gardner[2]

National Institute of Standards and Technology

Gaithersburg, Maryland, USA

[1]PTB, Braunschweig, Germany

[2] National Measurement Institute, Lindfield, Australia

ABSTRACT

Uncertainty estimates of detector spectral responsivity measurements are discussed. A model describes independent (uncorrelated) uncertainty components at a given wavelength based on the measurement equation for test detector calibrations. Systematic (correlated) components across wavelength are described briefly for spectrally integrated responsivity measurements. Propagation of the random and systematic components is discussed.

1. INTRODUCTION

The uncertainty analysis of spectral responsivity measurements should follow the "Guide to the Expression of Uncertainty in Measurement" (GUM) [1]. A Technical Report within the TC2-48 Technical Committee of the International Committee on Illumination (CIE) is being prepared to make recommendations for spectral responsivity measurements of detectors, radiometers, and photometers. The goal of the present work is to prepare a chapter for the CIE TC2-48 Technical Report on the uncertainty determination of spectral responsivity measurements using the GUM recommendations.

Starting with the measurement equation, a model is described to evaluate the radiant power responsivity determination of a test detector at a given wavelength. Uncertainty in the measured value arises from the uncertainty of the spectral responsivity of the standard (reference) detector, and from uncertainties in the elements of the correction factor used to evaluate the radiant power responsivity of the test detector from the measurement (calibration) transfer. Following the description of contributions that are independent of wavelength, systematic (correlated) uncertainty components in the reference detector are described and propagation of the components will be discussed for spectrally integrated measurements.

2. THE MEASUREMENT PROCESS

The relative standard uncertainty of spectral responsivity values can be derived from the measurement equation of the detector substitution method. The substitution method transfers the radiometric responsivity of a standard detector to a test detector that has similar properties to the standard detector. When the standard detector measures the constant input flux (the total radiant power in the incident beam) Φ, its output signal I_S is:

$$I_S = s_S \cdot \Phi \qquad (1)$$

where s_S is the known responsivity of the standard detector. In the following step, the test detector is substituted for the standard detector and measures the same radiant power in the same arrangement:

$$I_T = s_T \cdot \Phi \qquad (2)$$

where I_T is the output signal of the test detector and s_T is the unknown radiant power responsivity of the test detector that can be calculated from Eqs. (1) and (2):

$$s_T = s_S \cdot \frac{I_T}{I_S} \qquad (3)$$

When a monitor detector is used to decrease fluctuations in the radiant power, the modified measurement equation will be:

$$s_T = s_S \cdot \frac{I_T / I_{MT}}{I_S / I_{MS}} = s_S \cdot \frac{R_T}{R_S} \qquad (4)$$

where instead of the electrical output signals I_T, I_S their ratios to the simultaneously measured monitor output signals I_{MT}, I_{MS} are used: $R_T = I_T / I_{MT}$, $R_S = I_S / I_{MS}$.

3. MODEL AT A GIVEN WAVELENGTH

In Eq. (4) the radiant power responsivity of the test detector is calculated as a product of at least three factors. If we ignore the measurement errors of the electrical signals, the relative standard uncertainty in the spectral radiant power responsivity of the test detector can be written as

$$u_{\mathrm{rel}}(s_{\mathrm{T}}) = \left[u_{\mathrm{rel}}^2(s_{\mathrm{S}}) + u_{\mathrm{rel}}^2(R_{\mathrm{T}}) + u_{\mathrm{rel}}^2(R_{\mathrm{S}})\right]^{1/2} \quad (5)$$

In the brackets, the first contribution is the relative standard measurement uncertainty calculated from the certificate of the reference detector and the operational conditions during transfer. The other two contributions come from the measurement transfer.

Note that at present, calibration certificates do not report correlation coefficients or uncorrelated and correlated components separately. While measurements at the one wavelength do not depend on these correlations, they become important when combining values at different wavelengths in spectrally integrated components; this is discussed in Section 4 below.

The effective radiant power responsivity s_{S} of the reference detector requires a small correction for a possible temperature deviation ΔT (relative to the temperature during calibration), and for a relative non-uniformity β_{S}:

$$s_{\mathrm{S}} = s'_{\mathrm{S}} \cdot (1 + \alpha_{\mathrm{S}} \cdot \Delta T + \beta_{\mathrm{S}}) \quad (6)$$

where α_{S} is the relative temperature coefficient for the responsivity. The relative standard measurement uncertainty of the radiant power responsivity s'_{S} of the reference detector is calculated from the expanded measurement uncertainty $U(s'_{\mathrm{S}})$ and the coverage factor $(k = 2)$ given in the calibration certificate when the responsivity s'_{S} was measured at temperature T_{C} and is given by

$$u_{\mathrm{rel}}(s'_{\mathrm{S}}) = U(s'_{\mathrm{S}})/(2s'_{\mathrm{S}}). \quad (7)$$

The temperature deviation $\Delta T = T_{\mathrm{S}} - T_{\mathrm{C}}$ is calculated from the temperature T_{C} (nominal value, no uncertainty contribution) and the measured ambient temperature T_{S} during use. Often the standard deviation $s(T_{\mathrm{S}})$ of repeated thermometer readings is smaller than its resolution, which is an interval $\pm res(T_{\mathrm{S}})$ with rectangular probability distribution. The temperature is measured with the thermometer certified with an (absolute) expanded measurement uncertainty $U(T_{\mathrm{S}}) = \delta T$ for a $(k = 2)$ coverage interval and gives the standard variance $(\delta T/2)^2$. The standard variance of the temperature deviation from the three contributions is:

$$u^2(\Delta T) = \left(\frac{\delta T}{2}\right)^2 + s^2(T_{\mathrm{S}}) + \frac{res^2(T_{\mathrm{S}})}{3} \quad (8)$$

The relative temperature coefficient α_{S} is measured separately and the value can be determined with an associated relative standard measurement uncertainty $u(\alpha_{\mathrm{S}})$ or taken from the related literature.

Finally, the relative non-uniformity β_{S} has a zero value (it is included in the responsivity value) but its variation contributes to the responsivity measurement uncertainty within an interval $\pm\Delta s_{\mathrm{S}}$, having a rectangular probability distribution. Therefore, the standard variance is calculated as $u^2(\beta_{\mathrm{S}}) = \Delta s_{\mathrm{S}}^2/3$.

After these preparations and the two assumptions $1 >> |\alpha_{\mathrm{S}} \cdot \Delta T| + |\beta_{\mathrm{S}}|$ and no correlations between the quantities, the relative combined variance associated to the responsivity of the standard detector can be written:

$$u_{\mathrm{rel}}^2(s_{\mathrm{S}}) = u_{\mathrm{rel}}^2(s'_{\mathrm{S}}) + u^2(\beta_{\mathrm{S}}) + \alpha_{\mathrm{S}}^2 \cdot u^2(\Delta T) + u^2(\alpha_{\mathrm{S}}) \cdot \Delta T^2 \quad (9)$$

If the temperature at the time of measurement is the same as that at the time of calibration, $\Delta T = 0$. In this case, from second order terms, the factor ΔT^2 has to be replaced by the associated standard variance $u^2(\Delta T)$.

For the test detector, similar contributions have to be taken into consideration. The corrected value of the radiant power responsivity s'_{T} of the test detector can be calculated for the ambient temperature T_{C} and for zero contribution of its own non-uniformity:

$$s_{\mathrm{T}} = s'_{\mathrm{T}} \cdot (1 + \alpha_{\mathrm{T}} \cdot \Delta T + \beta_{\mathrm{T}}) \quad (10)$$

Similarly to the considerations before, the associated relative standard measurement uncertainty of s_T can be determined assuming small corrections $1 >> |\alpha_T \cdot \Delta T| + |\beta_T|$ and no correlation. Then the relative variance can be written:

$$u_{rel}^2(s_T) = u_{rel}^2(s'_T) + u^2(\beta_T) + \alpha_T^2 \cdot u^2(\Delta T) + u^2(\alpha_T) \cdot \Delta T^2 \qquad (11)$$

The combination of Eq. (4) with the corrections in Eq. (6) and (10) give a new model for the radiant power responsivity evaluation:

$$s'_T = s'_S \cdot \left[\frac{1 + \alpha_S \cdot \Delta T + \beta_S}{1 + \alpha_T \cdot \Delta T + \beta_T}\right] \cdot \frac{I_T / I_{MT}}{I_S / I_{MS}} \qquad (12)$$

and taking the earlier assumptions into account (first order approach), the bracket is simplified to $[1 + (\alpha_S - \alpha_T) \cdot \Delta T + \beta_S - \beta_T]$. This approach directly shows that for very similar test and reference detectors the temperature and spatial uniformity errors will cancel out.

The four photocurrents in Eq. (12), also contribute to the measurement uncertainty. The assumptions for the photocurrent measurements are:

A. The same amplifier and Digital Voltmeter (DVM) are used for both the test and reference detectors assuming that the gain and range settings are valid for both photocurrents.

B. With the test detector in place and shutter open, a series of $n \geq 20$ repeated readings are taken simultaneously for both the test (I'_T) and the monitor (I'_{MT}) detectors.

C. With the above conditions (especially for the amplifier gain and DVM range) but with the shutter closed, "dark measurements" (I'_{T0}, I'_{MT0}) are taken.

D. With the reference detector in place and shutter open, n repeated readings are taken simultaneously for both the reference (I'_S) and monitor (I'_{MS}) detectors.

E. Under the same conditions, but with the shutter closed, "dark measurements" (I'_{S0}), (I'_{MS0}) are taken.

Mean values like $\overline{I'_T}$ and its related standard deviations of the mean $s\left(\overline{I'_T}\right)$ are calculated for all the measured quantities and the two ratios: $R'_T = I'_T / I'_{MT}$; $R'_S = I'_S / I'_{MS}$. In principle, instead of these ratios, the dark current corrected ratios should be averaged. However, if the dark current is small and the measurement setup is fairly stable, then the measurement uncertainty evaluation can be simplified:

$$R_T = \frac{1}{n} \sum_i \frac{I'_{T,i} - \overline{I'_{T0,i}}}{I'_{MT,i} - \overline{I'_{MT0,i}}} \approx \overline{R'_T} \cdot \left(1 - \frac{\overline{I'_{T0}}}{\overline{I'_T}} + \frac{\overline{I'_{MT0}}}{\overline{I'_{MT}}}\right) \qquad (13)$$

The ratio for the reference detector readings can be calculated similarly. Finally, the double ratio in Eq. (4) will be:

$$\frac{R_T}{R_S} \approx \frac{\overline{R'_T}}{\overline{R'_S}} \cdot \left(1 + \frac{\overline{I'_{S0}}}{\overline{I'_S}} - \frac{\overline{I'_{T0}}}{\overline{I'_T}} + \frac{\overline{I'_{MT0}}}{\overline{I'_{MT}}} - \frac{\overline{I'_{MS0}}}{\overline{I'_{MS}}}\right) \qquad (14)$$

Usually, there are four contributions to the measurement uncertainty from a series of repeated readings of a radiometer: the standard deviations of the two mean values for light and dark readings; the calibration factor, that cancels out in Eq.(14); and the resolution $\pm \Delta I$ of the DVM. For all photocurrents, the standard measurement uncertainties are calculated similarly:

$$u\left(\overline{I'_{S0}}\right) = \sqrt{s^2\left(\overline{I'_{S0}}\right) + \left(\Delta I / \sqrt{3}\right)^2} \qquad (15)$$

If a common amplifier is used for both the standard and the test detectors, then the currents $\overline{I'_{S0}}, \overline{I'_{T0}}$ for dark (and offset) measurements are the same, as are their measurement uncertainties. As they are determined in a series, one after the other, there is no statistical correlation, even if recorded by the same amplifier.

Finally, the model to evaluate the radiant power responsivity of the test detector is calculated from the combination of Eqs. (12) and (14) with the higher orders omitted:

$$s'_T = s'_S \cdot \frac{\overline{R'_T}}{\overline{R'_S}} \cdot corr$$

$$corr = \left[\begin{array}{l} 1 + (\alpha_S - \alpha_T) \cdot \Delta T + \beta_S - \beta_T + \\ \dfrac{\overline{I'_{S0}}}{\overline{I'_S}} - \dfrac{\overline{I'_{T0}}}{\overline{I'_T}} + \dfrac{\overline{I'_{MT0}}}{\overline{I'_{MT}}} - \dfrac{\overline{I'_{MS0}}}{\overline{I'_{MS}}} \end{array}\right] \qquad (16)$$

All the contributions in the brackets are very small corrections and the associated relative standard measurement uncertainty is often negligible. In a reduced form, the relative standard measurement uncertainty associated to the radiant power responsivity is combined from four dominant contributions:

$$u_{\text{rel}}\left(s'_{\text{T}}\right)=\sqrt{u_{\text{rel}}^{2}\left(s'_{\text{S}}\right)+u_{\text{rel}}^{2}\left(\bar{R}'_{\text{T}}\right)+u_{\text{rel}}^{2}\left(\bar{R}'_{\text{S}}\right)+u_{\text{rel}}^{2}\left(corr\right)} \quad (17)$$

The principles in the measurement equation of Eq. (16) are valid for each single wavelength λ. Accordingly, all quantities are wavelength dependent:

$$s'_{\text{T}}\left(\lambda\right)=s'_{\text{S}}\left(\lambda\right)\cdot\frac{\overline{R'_{\text{T}}}\left(\lambda\right)}{\overline{R'_{\text{S}}}\left(\lambda\right)}\cdot corr\left(\lambda\right) \quad (18)$$

The correction factor includes contributions (originating from the measurement transfer) from temperature deviation ΔT, relative temperature coefficient of responsivities $\alpha_{\text{S}},\alpha_{\text{T}}$, relative spatial non-uniformity of responsivities $\beta_{\text{S}},\beta_{\text{T}}$, and dark (offset) currents $\overline{I'_{\text{S0}}},\overline{I'_{\text{T0}}},\overline{I'_{\text{MT0}}},\overline{I'_{\text{MS0}}}$ of the standard, test, and monitor detectors, all of which are independent of wavelength (or their changes are very small between neighbouring wavelengths where the measurement transfer is made).

The ratios $\overline{R'_{\text{T}}}(\lambda),\overline{R'_{\text{S}}}(\lambda)$ are formed from output signals $I_{\text{T}}(\lambda)$, $I_{\text{S}}(\lambda)$ with simultaneously measured monitor output signals $I_{\text{MT}}(\lambda)$, $I_{\text{MS}}(\lambda)$ using the correlation between the related photocurrents to eliminate noise from possible fluctuations of the source. Usually this is totally independent of wavelength.

Typically, the contributions for the determination of any single responsivity value and the associated uncertainty, are uncorrelated (at the given wavelength where the measurement transfer is made). These significant independent uncertainty components are added in quadrature. In case of independent variables, their covariances are zero. The quadrature sum can be applied to all random components at a given wavelength.

One component that may correlate the test and standard measurements is that of wavelength uncertainty. Random wavelength setting errors are correlated between test and standard measurements if the wavelength is set and the test and standard detectors measured; they are not correlated if the spectral range is swept independently for the test detector and standard detectors.

4. SPECTRALLY INTEGRATED RESPONSIVITY

The values representing the responsivity function $s'_{\text{S}}(\lambda)$ listed in a calibration certificate may be calculated from realizations totally independent for the different wavelengths. In this case, the uncertainty components may not be correlated. However in practice, the responsivity function of the reference detector – usually a trap detector calibrated with a laser beam at a limited number of wavelengths - is interpolated based on the physical knowledge (quantum efficiency) or mathematical smoothness and results in correlations. Similarly, the responsivity functions of detectors, calibrated with a trap detector, are corrected by mathematical procedures for wavelength setting, for errors due to output slit function or to smooth the values from wavelength to wavelength. All these procedures produce function values with uncertainties that are correlated. The correlations do not affect the individual values but become important if spectrally integrated quantities are calculated. Calibration transfer from the primary standard radiometer can also introduce correlated uncertainties to a working reference standard, due to systematic offsets or scaling factors common to all measurement wavelengths. Calibration certificates usually state the total uncertainty at a given wavelength and rarely distinguish between effects that are random and systematic (correlated) between wavelengths.

At the highest level (primary or secondary calibration laboratory) measurements, when combining spectral values over different wavelengths, it is usually the systematic components that determine the measurement uncertainty. For example. in a photodiode radiant power responsivity calibration, such systematic components (at all wavelengths) are the absolute responsivity of the reference detector, repeatability with beam position, and photodiode amplifier gain.

The integrated responsivity of a detector when it measures a broad-band light-source can be written as

$$R = \sum_n s_n P_n \qquad (19)$$

where P_n is the spectral power distribution of the light-source and s_n is the measured spectral power responsivity of the detector all at the *n*th (equally-spaced) wavelength. The detector is usually optically filtered to obtain a known spectral power responsivity function in a well defined wavelength interval.

The relative uncertainty of R has to be calculated. Uncertainties in s_n are independent of those in P_n and hence

$$u^2(R) = \sum_n \left(s_n{}^2 u^2(P_n) + P_n{}^2 u^2(s_n) \right) \qquad (20)$$

Often the spectral distribution of the source is a defined quantity (e.g. CIE Standard Illuminant A) and its values carry no uncertainty. In the following discussion we assume that source uncertainty is negligible; if required, the principles used to propagate uncertainties from the spectral responsivity values to that of the integrated response are identical for those of the source uncertainties.

As seen in Eqs. (3) and (4), determination of spectral responsivity is a transfer from a reference standard to the device under test,

$$s_{T,n} = s_{S,n} t_n \qquad (21)$$

where n denotes wavelength and t is the transfer ratio at the nth wavelength. Uncertainties in spectral responsivity of the test detector arise from those of the standard and from the transfer process; these are treated independently and then added in quadrature.

4.1 Uncertainties of the standard detector

From Eq. (21),

$$u(s_{T,n}) = t_n u(s_{S,n}) = \frac{s_{T,n}}{s_{S,n}} u(s_{S,n}) \qquad (22)$$

where the uncertainties in spectral responsivity of the reference detector are obtained from the calibration certificate. These may be complex, depending on the method of generating the primary standard. In such cases, a full propagation using covariances between the reference values is required [1,2] – these must be sought from the supplier of the reference standard.

Some certificates provide total random component uncertainties and total correlated uncertainties at each wavelength. Where the ratio (systematic component : total uncertainty) of these components is approximately constant through the spectral range of interest, the correlation coefficient r is also approximately constant and given by the square of the ratio and effective uncertainties that are random and fully-correlated between wavelengths are readily determined. The effective uncertainty in spectral responsivity that is random between wavelengths is then given by

$$u_R(s_n) = \sqrt{1-r}\;\; u(s_n) \qquad (23)$$

and the total contribution of random effects to the uncertainty in the integrated spectral quantity is given by

$$u_R{}^2(R) = \left(1-r\right) \sum P_n{}^2 u^2(s_n) \qquad (24)$$

The effective uncertainty in spectral responsivity that is fully correlated between wavelengths is given by

$$u_S(s_n) = \sqrt{r}\;\; u(s_n) \qquad (25)$$

The total contribution of the systematic effects is given by the linear sum:

$$u_S(R) = r \sum P_n u(s_n) \qquad (26)$$

Alternatively, where the systematic and random components are given at each wavelength, or their wavelength dependence is given as a scaling of some parameter, each systematic component may be propagated separately as a linear sum, and the total random components propagated in quadrature.

The total uncertainty contribution of the reference standard is the quadrature sum of the systematic and random components.

A further complication is that spectral responsivity values for the reference standard may require interpolation to the wavelength values of the measurement. Interpolation introduces correlations and these must be taken into account when propagating the uncertainties of the standard detector. A simple process that avoids these complications is to interpolate the transfer ratio to the wavelengths at which the reference standard is calibrated and propagate uncertainties in the integral sum (including the change in wavelength spacing) using only those values [3].

4.2 Uncertainties of transfer process

Each effect contributing to uncertainty in the transfer can be classified as independent and either random between wavelengths or fully-correlated between wavelengths. These effects can also be separately determined for the numerator or denominator of the transfer ratio, as shown in Eqs. (3) and (4), and combined either linearly or in quadrature depending on whether the effects are correlated or random between test and reference signals at the one wavelength.

Once the uncertainty in the transfer ratio and hence its uncertainty in spectral responsivity

$$u_e(s_{T,n}) = s_{S,n} u_e(t_n) \qquad (27)$$

is determined for each effect at each wavelength, its contribution to the integral sum is calculated. For random components,

$$u_e^{\,2}(R) = \sum P_n^{\,2} u_e^{\,2}(s_n) \qquad (28)$$

For systematic components, uncertainties in the values of spectral responsivity at different wavelengths are generally positively correlated, and the contribution to the integral sum is given by

$$u_e(R) = \sum P_n u_e(s_n) \qquad (29)$$

However some effects (e.g. wavelength offsets) can produce correlations that are positive or negative between wavelength pairs. These effects are properly handled by attaching a sign, that of the sensitivity coefficient for the effect, to the uncertainty in the transfer ratio. For example, consider the transfer ratio shown in Eq. (21). In the presence of a wavelength uncertainty fully correlated between test and reference, the signed uncertainty in the transfer ratio test:standard signals at the *n*th wavelength is given by

$$u_{e,s}(t_n) = \left(\frac{1}{R_{S,n}} \frac{\partial R_{T,n}}{\partial \lambda} - \frac{R_{T,n}}{R_{S,n}^{\;2}} \frac{\partial R_{S,n}}{\partial \lambda} \right) u(\lambda) \qquad (30)$$

The signed uncertainty in spectral responsivity for this effect at the *n*th wavelength is then given by combining Eqs. (27) and (30):

$$u_{e,s}(s_{T,n}) = \frac{s_{R,n}}{R_{S,n}} \left(\frac{\partial R_{T,n}}{\partial \lambda} - t_n \frac{\partial R_{S,n}}{\partial \lambda} \right) u(\lambda) \qquad (31)$$

This expression can be positive or negative depending on the slopes of the test and reference signals with respect to wavelength. The propagated uncertainty for a wavelength offset applicable at all wavelengths, including these mixed correlations, is found by the linear sum

$$u_e(R) = \sum P_n u_{e,s}(s_{T,n}) \qquad (32)$$

Detailed discussions and examples for uncertainty contributions of different correlated combinations can be found in [2].

5. CONCLUSION

When reporting spectral responsivity measurement uncertainties, the total uncertainty contribution of effects systematic over wavelengths should be given at each wavelength, instead of the total uncertainty as is the usual practice.

REFERENCES

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